B.Sc. 4th Semester (Honours) Examination, 2020-21

PHYSICS

Course ID: 42411 **Course Code: SH/PHS/401/C-8/T-8**

Course Title: Mathematical Physics-III

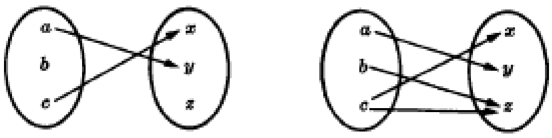
Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the right hand side margin indicate marks.
Candidates are required to give their answers in their own words
as far as practicable.

Section - I

1. Answer any ***five*** questions: 1x5=5

(a) State which mapping from A= {a, b, c} to B= {x, y, z} defines a function and why?



(b) Find the norm and normalized vector form of (1, i).

(c) Define 'Binary operation' in abstract algebra.

(d) Find Laplace transform of (1+ $\cos 2t$)

(e) State Cayley Hamiltonian theorem.

(f) State convolution theorem in connection with the Laplace transformation.

(g) Find the Fourier transform of f(x) = $\begin{bmatrix} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{bmatrix}$

(h) A 3x3 matrix satisfies the equation $M^2 - 3M + 2I = 0$ (I is the identity matrix). Find out the determinant of the matrix if its trace is 6.

Section – II

2. Answer any ***two*** questions: 5x2=10

(a) Obtain a set of four orthonormal vectors by the Schmidt's method from the vectors **U1**=(1, 1, 0, 1), **U2**=(2, 0, 0, 1), **U3**=(0, 2, 3, -2), **U4** = (1, 1, 1, -5).

(b) Find the inverse Laplace transform of $\frac{2s-5}{9s^2-25}$.

(c) Using matrix method solve the differential equation

$\dot{x} = 6x + 5y$ and $\dot{y} = x + 2y$

P.T.O.

(d) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & for\ |x| \quad < 1 \\ 0 & for\ |x| \quad > 1 \end{cases}$

Section – III

3. Answer any ***one*** question: 10x1=10

(a) (i) If Laplace transform of F(t), i.e., L{F(t)} = f(s) then show that $L\left\{\int_0^t F(u)du\right\} = \frac{f(s)}{s}$. (ii) A semi-infinite transmission line of negligible inductance and leakage per unit length has its voltage and current equal to zero. A constant voltage is applied at its sending end (x=0) at time t=0. Using Laplace transform find the voltage and current that flowing in the transmission line at any point (x>0) and at any instant t. You may use the equations of transmission line $\frac{\partial v}{\partial x} = -Ri$; $\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t}$ [symbols have their usual meanings] 3+7

(b) Using Fourier integral representation, show that $\int_0^{\propto} \frac{\lambda sin\lambda x}{(\lambda^2+\alpha^2)(\lambda^2+\beta^2)} d\lambda = \frac{\pi}{2}\frac{(e^{-\alpha x}-e^{-\beta x})}{(\beta^2-\alpha^2)}$. Hence, find the Fourier sine integral representation of $(e^{-x} - e^{-2x})$. Find the complex Fourier transform of the Dirac delta function $\delta(t-a)$. 5+2+3
